Series–C A–857–C

Roll No. [redacted]....

(Graph Paper)

Total No. of Questions–31] [Total No. of Printed Pages–11

A–857–C –XII–2318

MATHEMATICS

(English Version)

Time Allowed–3 Hours Maximum Marks–85

Candidates are required to give their answers in their own words as far as possible.

Marks allotted to each question are indicated against it.

Special Instructions :

(i) *You must write question paper series in the circle at the top left side of title page of your answer book.*

(ii) *While answering questions, you must indicate on your answer-book the same question no. as appears in your question paper.*

(iii) *Do not leave blank page/s in your answer-book.*

(iv) *Question Nos. 1 to 10 are of Multiple Choice Questions and are of 1 mark each. Question Nos. 11 to 14 are of 2 marks each. Question Nos. 15 to 26 are of 3½ marks each. Question Nos. 27 to 31 are of 5 marks each.*

(v) ***All questions are compulsory.***

(vi) *Internal choices have been provided in some questions.*

(vii) *Use of calculator is not permitted. You may ask for Logarithmic tables and graph paper if necessary/needed.*

1. The principal value of $\tan^{-1}(-\sqrt{3})$ is : 1

(a) $\frac{\pi}{2}$ (b) $-\frac{\pi}{3}$

(c) $\frac{\pi}{3}$ (d) $-\frac{\pi}{2}$

2. Let A be a nonsingular square matrix of order 3×3. Then $|\text{adj } A|$ is : 1

(a) $|A|$

(b) $|A|^3$

(c) $|A|^2$

(d) $|3A|$

3. The derivative of 5^x is : 1

(a) 5^x

(b) $\frac{5^x}{\log 5}$

(c) $5^x \log 5$

(d) None of these

4. On which of the following intervals is the function f given by $f(x) = x^{100} + \sin x - 1$ strictly decreasing? 1

(a) $(0, 1)$

(b) $\left(\frac{\pi}{2}, \pi\right)$

(c) $\left(0, \frac{\pi}{2}\right)$

(d) None of these

5. $\int e^x (f(x) + f'(x))\, dx$ is equal to : 1

(a) $e^x f'(x) + c$ (b) $e^x f(x) + c$

(c) $-e^x f'(x) + c$ (d) $-e^x f(x) + c$

6. The degree of differential equation

$$\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0 \text{ is :}$$ 1

(a) 4 (b) 1

(c) 2 (d) Not defined

7. The vectors $\vec{a}$ and $\vec{b}$ are perpendicular if : 1

(a) $\vec{a} \cdot \vec{b} = 0$ (b) $\vec{a} \cdot \vec{b} \neq 0$

(c) $\vec{a} \times \vec{b} = 0$ (d) $\vec{a} \times \vec{b} \neq 0$

8. Find $|\vec{a} - \vec{b}|$, if $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 4$ 1

(a) $\sqrt{3}$ (b) $\sqrt{2}$

(c) $\sqrt{5}$ (d) $\sqrt{7}$

9. If a line makes angles $\frac{\pi}{2}, \frac{3\pi}{4}$ and $\frac{\pi}{4}$ with x, y, z - axis, respectively then direction cosines of this line are : 1

(a) $\pm(1, 1, 1)$ (b) $\pm\left(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

(c) $\pm\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$ (d) $\pm\left(0, \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

10. If A and B are independent events, then : 1

(a) $P(A \cap B) = P(A) . P(B)$

(b) $P(A \cup B) = P(A) . P(B)$

(c) $P(A \cap B) = P(A) + P(B)$

(d) $P(A \cup B) = P(A) + P(B)$

11. Using elementary operations, find the inverse of

matrix $A=\begin{bmatrix}1 & 3\\ 2 & 7\end{bmatrix}$

or

For matrix

$$A=\begin{bmatrix}1 & 5\\ 6 & 7\end{bmatrix}$$

Verify that $A-A'$ is a skew symmetric matrix. 2

12. Examine the function given by

$$f(x)=\begin{cases}\frac{\sin x}{x}, x<0\\ x+1, x\geq 0\end{cases}$$ for continuity. 2

13. A balloon which always remains spherical, has a variable diameter $\frac{3}{2}(2x+1)$. Find the rate of change of its volume with respect to x. 2

14. Form the differential equation of the family of hyperbolas having foci on x-axis and centre at origin.

15. Find $g \circ f$ and $f \circ g$, if $f(x) = 8x^3$ and $g(x) = x^{\frac{1}{3}}$

3½

16. Solve $2 \tan^{-1}(\cos x) = \tan^{-1}(2 \operatorname{cosec} x)$

or

Express for following in the simplest form :

$\tan^{-1}\left(\frac{x}{\sqrt{a^2 - x^2}}\right), |x| < a$ 3½

17. Prove that $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4\,abc$ 3½

18. Differentiate $\sin(\{\tan^{-1}(e^{-x})\})$ w.r.t. x

or

Find $\frac{dy}{dx}$ if $xy = e^{(x-y)}$ 3½

19. Evaluate $\int \frac{5x}{(x+1)(x^2-4)} dx$ 3½

20. Evaluate $\int \frac{6x+7}{\sqrt{(x-5)(x-4)}} dx$ 3½

21. Evaluate $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$ 3½

22. Solve the differential equation :

$$(x+y)\frac{dy}{dx} = 1$$

or

Solve the differential equation :

$$(x-y)\,dy - (x+y)\,dx = 0.$$ 3½

23. Find x if the four points A (3, 2, 1), B (4, x, 5), C (4, 2, –2) and D (6, 5, –1) are coplanar. 3½

24. Find the vector and Cartesian equations of plane that passes through the point (5, 2, –4) and perpendicular to the line with direction ratios 2, 3, –1. 3½

25. A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event, 'the number is even' and B be the event, 'the number is red'. Are A and B independent? 3½

26. If pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability of two successes.

or

Give two independent events A and B such that $P(A) = 0.3$, $P(B) = 0.6$, find

(i) P (A and B)

(ii) P (A and not B). 3½

27. Solve the following equations by Matrix method :

$$2x + 3y + 3z = 5$$

$$x - 2y + z = -4$$

$$3x - y - 2z = 3$$

5

28. Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ time the radius of the base.

or

Find the point on the curve $y = x^3 - 11x + 5$ at which the tangent is : $y = x - 11$. 5

29. Find the area of region bounded by the curves : $x^2 = y$, the line $y = x + 2$ and the x-axis.

or

Using integration find the area of region bounded by the triangle whose vertices are (–1, 0), (1, 3) and (3, 2). 5

30. Find the shortest distance between the lines

$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ and

$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$

or

Find the equation of the plane that passes through three points (2, 5, –3), (–2, –3, 5) and (5, 3, –3).

5

31. Solve the following linear programming problem graphically.

Minimize $Z = -3x + 4y$

subject to the following constraints :

$$x + 2y \leq 8,$$

$$3x + 2y \leq 12,$$

$$x \geq 0,\ y \geq 0$$ 5

[Encl. Graph Pap